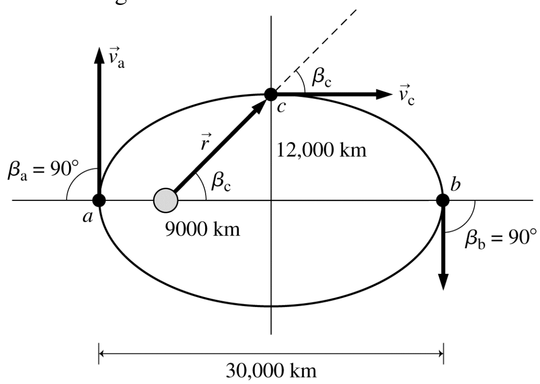
13.81. Model: The angular momentum of the satellite in the elliptical orbit is a constant.

Visualize:



Solve: **(a)** Because the gravitational force is always along the same direction as the direction of the moment arm vector, the torque $\tau = r \times F_g$ is zero at all points on the orbit.

(b) The angular momentum of the satellite at any point on the elliptical trajectory is conserved. The velocity is perpendicular to $\vec{r}$ at points a and b, so $\beta = 90°$ and $L = mvr$. Thus

$$L_b = L_a \Rightarrow mv_b r_b = mv_a r_a \Rightarrow v_b = \left(\frac{r_a}{r_b}\right) v_a$$

$$r_a = \frac{30{,}000 \text{ km}}{2} - 9{,}000 \text{ km} = 6.00 \times 10^6 \text{ m} \quad \text{and} \quad r_b = \frac{30{,}000 \text{ km}}{2} + 9{,}000 \text{ km} = 2.4 \times 10^7 \text{ m}$$

$$\Rightarrow v_b = \left(\frac{6.00 \times 10^6 \text{ m}}{2.4 \times 10^7 \text{ m}}\right)(8{,}000 \text{ m/s}) = 2000 \text{ m/s}$$

(c) Using the conservation of angular momentum $L_c = L_a$, we get

$$mv_c r_c \sin\beta_c = mv_a r_a \Rightarrow v_c = \left(\frac{r_a}{r_c}\right) v_a / \sin\beta_c \qquad r_c = \sqrt{(9000 \text{ km})^2 + (12{,}000 \text{ km})^2} = 1.5 \times 10^7 \text{ m}$$

From the figure, we see that $\sin\beta_c = 12{,}000/15{,}000 = 0.80$. Thus

$$v_c = \left(\frac{6.0 \times 10^6 \text{ m}}{1.5 \times 10^7 \text{ m}}\right)\frac{(8000 \text{ m/s})}{0.80} = 4000 \text{ m/s}$$